

# Study of the $K_{e4}$ ( $\pi^0\pi^0e^\pm\nu$ ) decay with NA48/2 @ CERN

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on behalf of the NA48/2 Collaboration

**KAON13**



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**University of Michigan , Ann Arbor, April 29-May 1**

# Outline

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- ❖ No need to repeat NA48/2 description of experimental setup & detector performances...
- ❖ The  $K_{e4}$  decay mode ( $\pi^0 \pi^0 e^\pm \nu$ )
- ❖ Selection and backgrounds
- ❖ Branching Ratio
- ❖ Form factor
- ❖ Summary

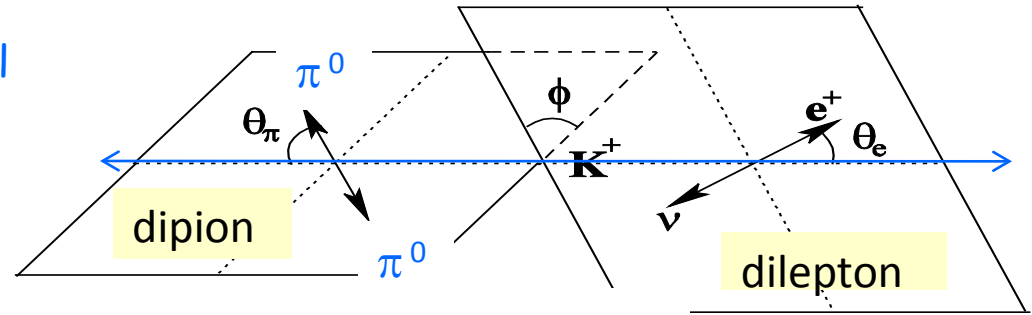
Preliminary!

# Ke4 (00) decays : a simpler formalism than Ke4(+)

Four-body final state with two identical particles, described by 3 kinematic variables:

$S_\pi (M_{\pi\pi}^2)$ ,  $S_e (M_{e\nu}^2)$ ,  $\cos\theta_e$

$$d^3\Gamma = \frac{G_F^2 |V_{us}|^2}{2(4\pi)^6 m_K^5} \rho(S_\pi, S_e) 2|X F_s|^2 \sin^2 \theta_e \times dS_\pi dS_e d\cos \theta_e.$$



Complex hadronic form factors  $F_1, F_2, F_3$  ( $F_4$ ) reduce to  $F_1$

Partial Wave expansion of  $F_1$  reduces to S-wave (no P-wave in the  $\pi^0 \pi^0$  system) if neglecting D-wave

Only 1 complex Form Factor

$$F = F_s e^{i\delta_s}$$

$\cos\theta_e$  distribution does not carry information

Map the distributions of the  $S_\pi, S_e$  variables in the two-dimensional space with 1 real Form Factor (may be energy dependent)

The fit parameter (real number) is only  $F_s$  determined in a grid of statistically independent (equi-populated) boxes in the  $(M_{\pi\pi}, M_{e\nu})$  plane

# Ke4 (00) BR and form factor: experiments and theory

Previous experiments had very low statistics (PDG 2012)

37 events from 3 experiments:  $BR = (2.2 \pm 0.4) 10^{-5}$  (18% rel. error)

214 events from KEK E470 (not considered):  $BR = (2.29 \pm 0.34) 10^{-5}$  (large syst.)

No form factor determination so far, just a relation between partial rate and a constant form factor value :

$$\Gamma = 0.8 |V_{us} \cdot F|^2 10^3 \text{s}^{-1}$$

Using the kaon mean life time  $(1.2380 \pm 0.00021) 10^{-8} \text{s}$ , it translates to  
 $|V_{us}| \cdot F = 1.49 \pm 0.13$  or  $F = 6.61 \pm 0.58$  for  $|V_{us}| = 0.2252 \pm 0.0009$

Theoretical predictions :

Isospin symmetry ( $m_u = m_d = 0, \alpha_{\text{QED}} = 0$ ) predicts a relation between rates  
 $\Gamma(K_{l4}^{+-}) = \frac{1}{2} \Gamma(K_{l4}^{0\pm}) + 2 \Gamma(K_{l4}^{00})$  (valid for lepton = e,  $\mu$ )  
 $K^\pm$  (2.4% now 0.8%)    $K^0$  (2.1%)    $K^\pm$  (18%)

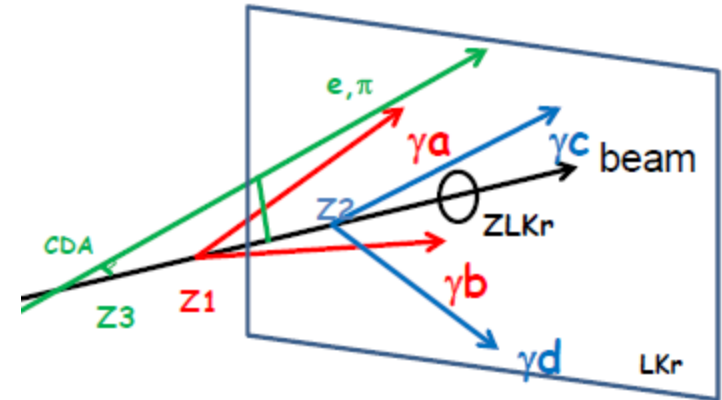
ChPT calculations  $O(p^2, p^4, p^6)$  from Bijmans Colangelo Gasser (NPB 427 (1994) 427) using available 1977 Ke4(+/-) form factors predict :

$$BR(K_{e4}^{00}) = (2.01 \pm 0.11) 10^{-5} (\sim 5\% \text{ precision})$$

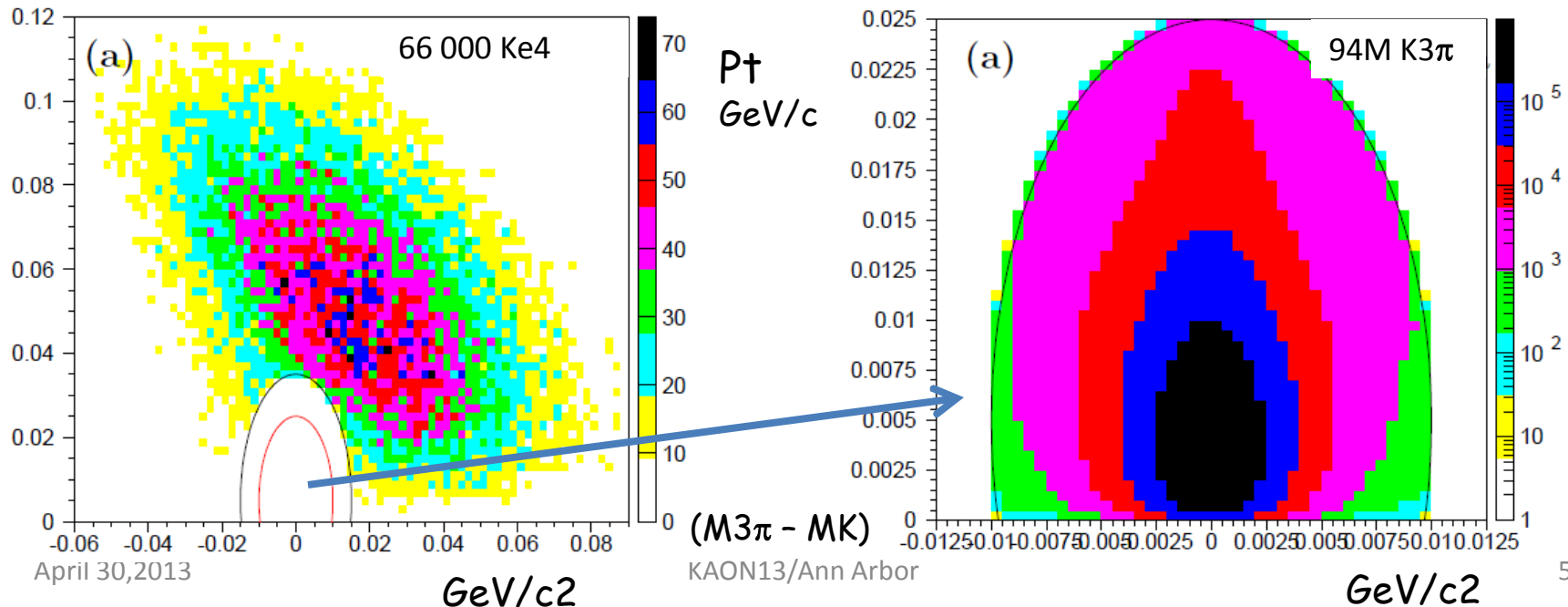
# Event selection

Final state reconstructed from 1 charged track and 4 photons forming two  $\pi^0$ s pointing to the same decay vertex

Reconstruct the  $\pi^\pm \pi^0 \pi^0$  invariant mass assuming the charged track to be a pion.



In the plane ( $M_{3\pi}$  - MPDG,  $p_T$ ),  $K_{e4}$  candidates (missing neutrino) are well separated from  $K_{3\pi}$  fully reconstructed events.



# Particle-ID and background contamination

## Normalization: requires pion-ID

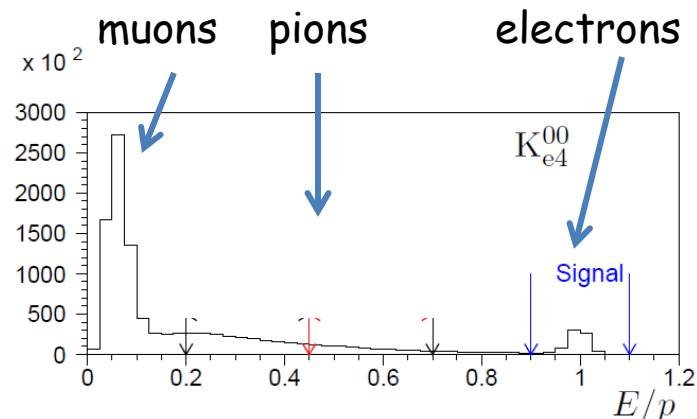
- pion momentum  $> 5 \text{ GeV}/c$
- no requirement of an associated cluster

94 M candidates

## Signal: requires electron-ID

- electron momentum  $> 5 \text{ GeV}/c$
- LKr Calorimeter cluster associated to track
- $E(\text{LKr})/p$  within  $[0.9, 1.1]$  + shower properties

66K candidates



## :Background in Signal region

- fake-electron ( $\pi^0 \pi^0 \pi^\pm$ ) measured with a data driven procedure
- real electron ( $\pi^0 \pi^0 \pi^\pm$  &  $\pi^\pm$  decay to  $e^\pm \nu$ ,  $\text{BR} = 1.23 \cdot 10^{-4}$ ) estimated from a dedicated modified simulation
- accidentals (photons or tracks) measured from data with relaxed timing cuts and using side bands

B/S+B

0.71 %

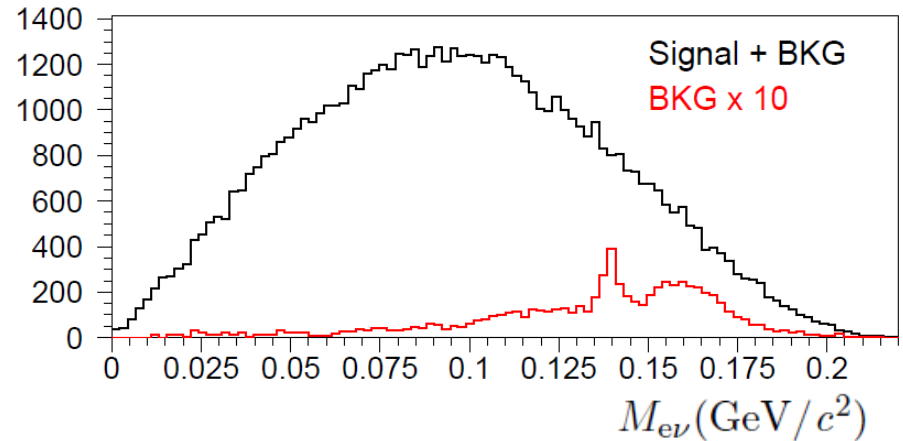
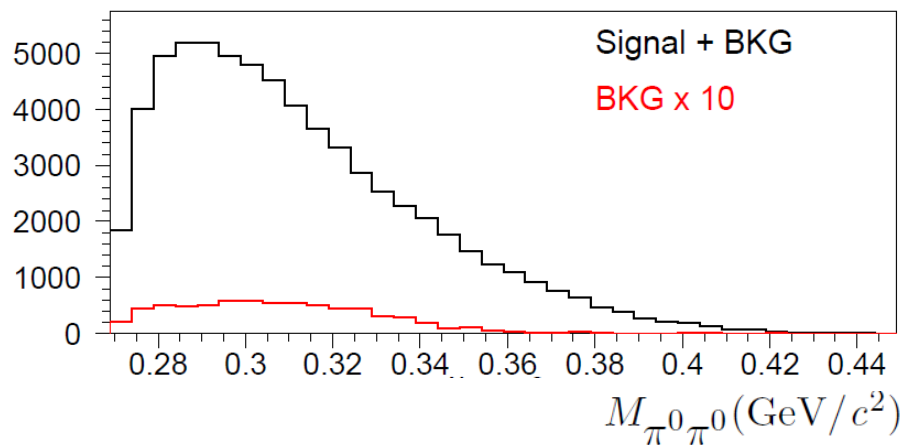
0.12 %

0.24 %

1.07 %

# Form factor measurement : principle

- Differential rate in the  $(S_{\pi}, S_e)$  plane is proportional to  $|F_S|^2$
- Subtract background in the 2d-plane

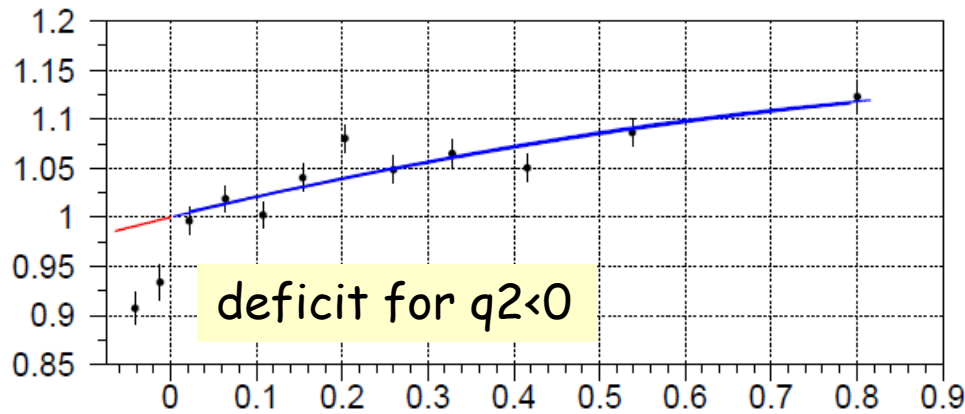


- Compare to the same distribution obtained from simulation including acceptance, resolution, trigger efficiency, radiative corrections (real photon emission at decay vertex) and kinematic factors but using a **constant form factor**
- switch to dimensionless variables:  $q^2 = S_{\pi}/4m_{\pi^+}^2 - 1$ ,  $S_e/4m_{\pi^+}^2$
- Define a grid of 10 equal population bins in  $q^2$  above  $q^2=0$  ( $2m_{\pi^+}$  threshold) and two equal population bins below (10 bins with 6000 events each, 2 bins with 3000 events each), 10 bins in  $S_e$  (300 or 600 events in 2d-bins)

$$M_{e\nu}(\text{GeV}/c)$$

# Form factor measurement : energy dependence

$(F_s/f_s)^2$



Ratio (Data / MC)  $\sim |F_s|^2$

focus on  $q^2$  dependence above  $q^2=0$

Normalization is adjusted in the fit to have a value 1 for  $q^2 = 0, S_e = 0$

$$(1 + f'_s/f_s q^2 + f''_s/f_s q^4)^2$$

Step 1:  $q^2$  dependence only

$$(1 + f'_s/f_s q^2 + \bar{f}''_s/\bar{f}_s q^4 + f'_e/\bar{f}_s S_e/4m_{\pi^+}^2)^2$$

Step 2 : include  $S_e$  dependence

Fit  $q^2$  1d-projection

$$f'_s/f_s = 0.102 \pm 0.031$$

$$f''_s/f_s = -0.040 \pm 0.037$$

$$\rho = -0.949$$

$$\text{chi}^2/\text{ndf} = 2.06 \text{ (5\%)}$$

Fit  $q^2$  in 2d-plane

$$f'_s/f_s = 0.110 \pm 0.032$$

$$f''_s/f_s = -0.051 \pm 0.039$$

$$\rho = -0.954$$

$$\text{chi}^2/\text{ndf} = 1.22 \text{ (8\%)}$$

Fit 2d-plane

$$f'_s/f_s = 0.136 \pm 0.033$$

$$f''_s/f_s = -0.060 \pm 0.039$$

$$f'_e/f_s = 0.110 \pm 0.024$$

$$C = \begin{matrix} -0.946 & 0.189 \\ & -0.063 \end{matrix}$$

$$-0.063$$

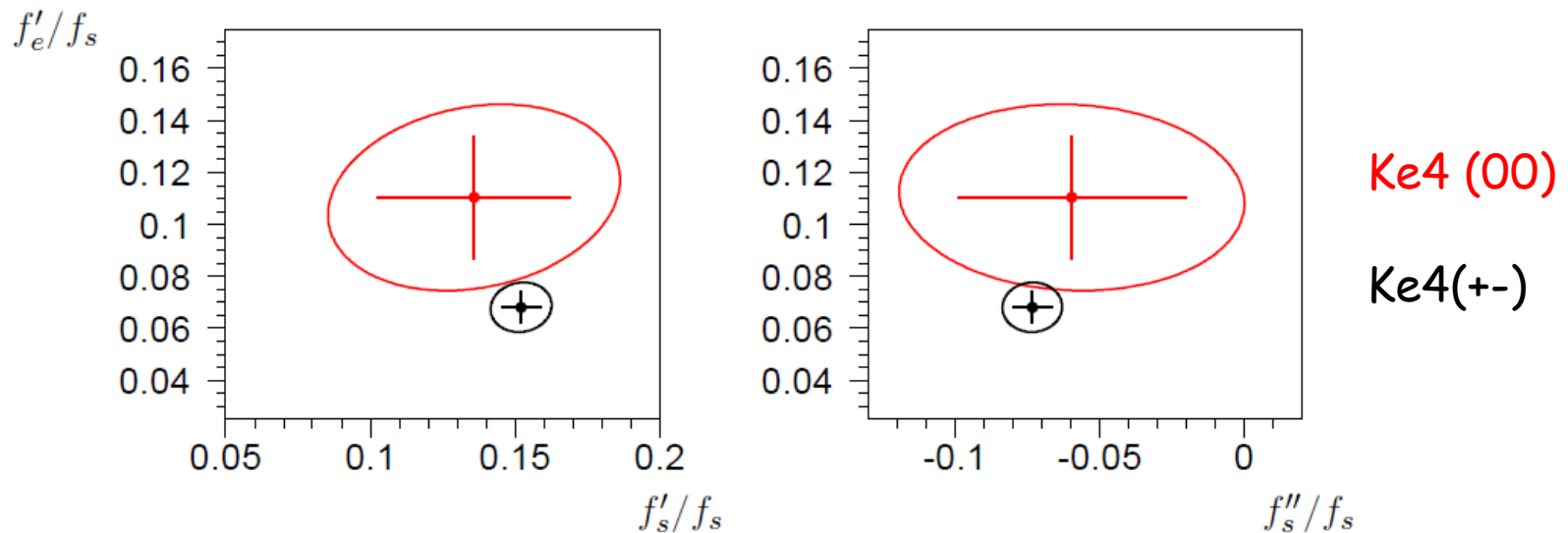
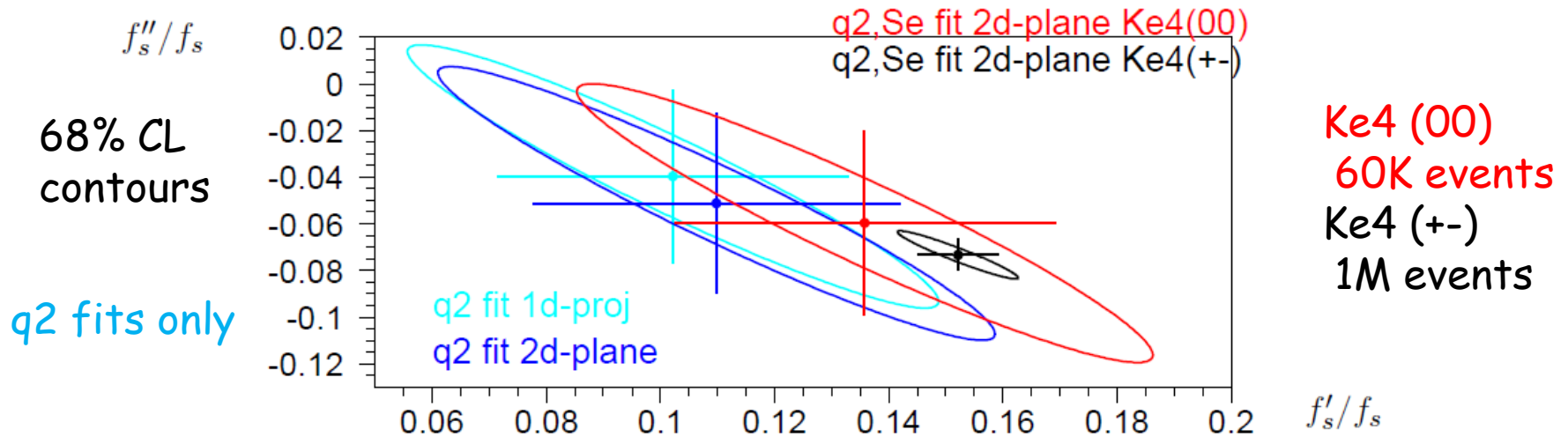
$$\text{chi}^2/\text{ndf} = 0.97 \text{ (54\%)}$$

The improved  $\text{chi}^2$  supports the extra fit parameter  $\rightarrow$



# Form factor energy dependence

Are the various results statistically consistent? **YES**, also with the charged Ke4 mode



# Form factor systematics and results

Source	$f'_s/f_s$	$f''_s/f_s$	$f'_e/f_s$
fit procedure	—	—	—
reconstruction	—	—	—
trigger corrections	—	—	—
acceptance control	0.002	0.002	0.001
background control	0.012	0.013	0.022
electron-id	0.008	0.007	0.002
total systematics	0.015	0.015	0.022
statistical error	0.033	0.040	0.024

Preliminary!

$$f'_s/f_s = 0.136 \pm 0.033_{stat} \pm 0.015_{syst}$$

$$f''_s/f_s = -0.060 \pm 0.039_{stat} \pm 0.015_{syst}$$

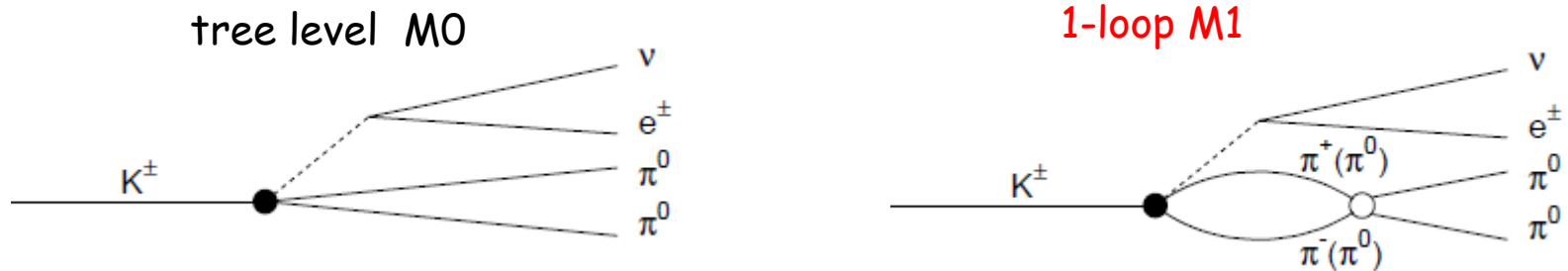
$$f'_e/f_s = 0.110 \pm 0.024_{stat} \pm 0.022_{syst}$$

Chi2/ndf = 0.97 (54% prob)

	$f''_s/f_s$	$f'_e/f_s$
$f'_s/f_s$	-0.946	0.189
$f''_s/f_s$		-0.063

# Form factor below $q^2 = 0$

The 10% drop (cusp-like) for  $q^2 < 0$  can be interpreted as final state charge exchange scattering in the  $Ke4(+)$  mode :



Follow papers by Cabibbo (PRL 93 (2004)) and Cabibbo-Isidori (JHEP 03 (2005)) to write the amplitudes :

$$M_0 = f_s(1 + a q^2 + b q^4 + c S_e/4m_{\pi^+}^2),$$

$$M_1 = -2/3 (a_0^0 - a_0^2) f_s \sigma_\pi,$$

$$q^2 = S_\pi/4m_{\pi^+}^2 - 1 \text{ and } \sigma_\pi = \sqrt{1 - 4m_{\pi^+}^2/S_\pi} = \sqrt{|q^2/(1 + q^2)|}$$

above threshold ( $q^2 > 0$ ):  $|M|^2 = |M_0 + iM_1|^2 = M_0^2 + M_1^2$

below threshold ( $q^2 < 0$ ):  $|M|^2 = |M_0 + M_1|^2 = M_0^2 + M_1^2 + 2M_0 M_1$

**$M$  is reduced as  $M_1 < 0$**

# Playing with amplitudes

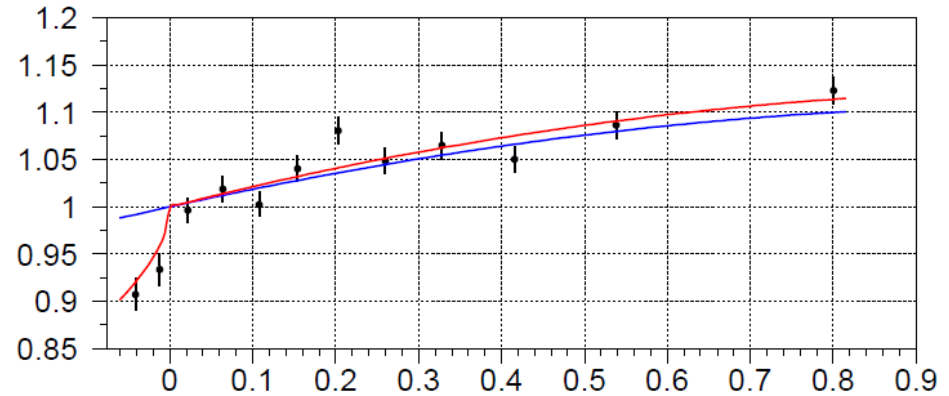
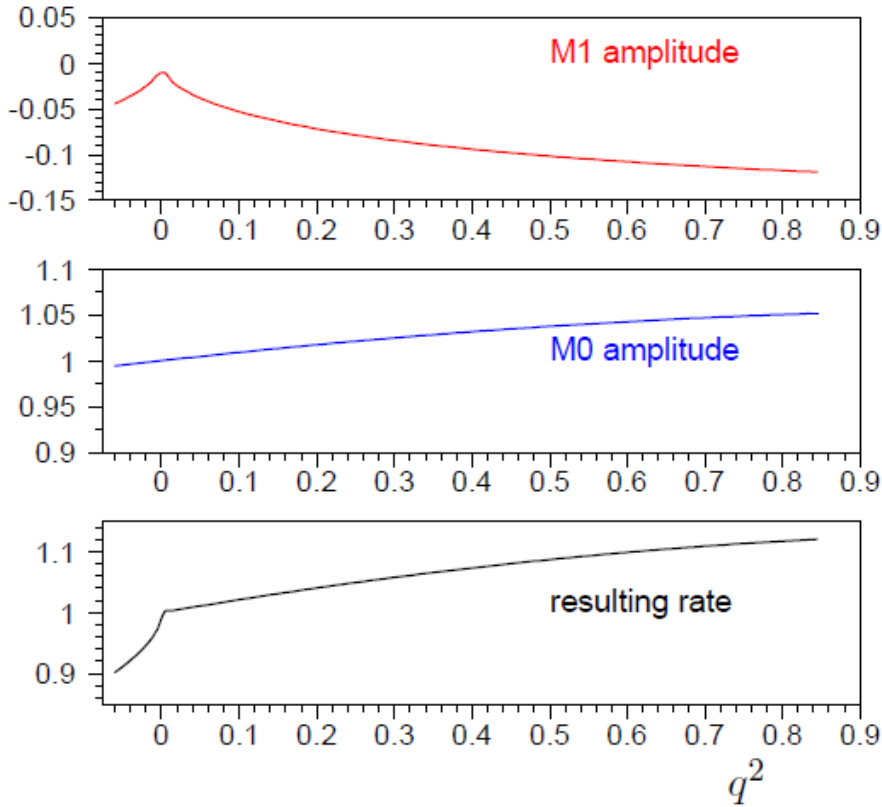
Preliminary!

Simple exercise :

- M1 and M0 add up to  $|M|^2$
- reproduce qualitatively the deficit below  $q^2 = 0$

Replaying the fit to the  $(q^2, S_e)$  plane, including the known M1 contribution for  $q^2 > 0$ , one gets **different parameter values** with same errors and correlations

$$\begin{aligned} f'_s / f_s &= 0.123 \pm 0.034_{stat} \\ f''_s / f_s &= -0.054 \pm 0.040_{stat} \\ f'_e / f_s &= 0.111 \pm 0.024_{stat} \end{aligned}$$



Extending the fitted function to  $q^2 < 0$ , reproduces the observed distribution (not a fit to the  $q^2 < 0$  distribution !)

# Branching ratio measurement and uncertainties

$$BR(K_{e4}) = (N4 - Nbkg) / N3 \times A3 / A4 \times \varepsilon(K3\pi) / \varepsilon(K_{e4}) \times BR(K3\pi)$$

<b>N4</b> Signal events	66000	(1.761 ± 0.022) %
<b>Nbkg</b> background events	708 (~1% relative)	
<b>N3</b> normalization events $K3\pi$ ( $\pi^\pm \pi^0 \pi^0$ )	$94 \times 10^6$	
<b>A3</b> normalization Acceptance	4.05%	
<b>A4</b> signal Acceptance	1.92%	
$\varepsilon(K3\pi)$ normalization trigger eff	97.4%	
$\varepsilon(K_{e4})$ signal trigger eff	96.1%	

Preliminary!

		Source	Contribution (%) to BR uncertainty
Systematic error	0.38%	Acceptance stability with cuts	0.15 (not final)
		Form factor uncertainty	0.13
		Background evaluation	0.02
		Accidental activity	0.02
		Trigger cut	0.08
		Particle identification	0.25
		Radiative events modeling	0.17
		Simulation statistics	0.09
		Trigger efficiency	0.03
External error	1.25%	External error	1.25
Statistical error	0.39%	Statistical error	0.39

# Branching ratio result

combination of 10 statistically independent samples  
 (each with stable data taking conditions)

$$\Gamma(\text{Ke}400)/\Gamma(\text{K}3\pi) = (1.468 \pm 0.008_{\text{exp}}) 10^{-3}$$

0.6% relative uncertainty

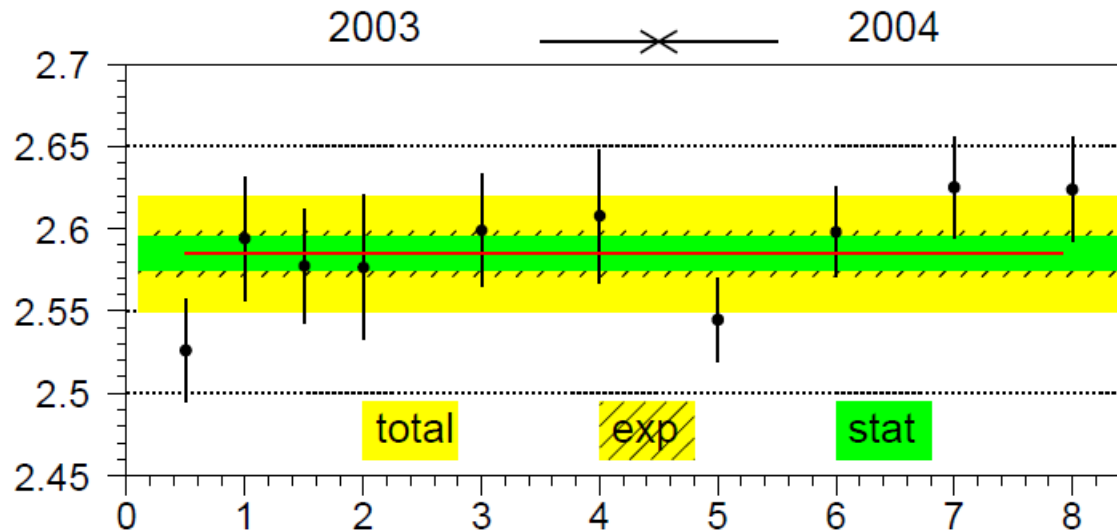
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$$\text{BR}(\text{Ke}400) = (2.585 \pm 0.010_{\text{stat}} \pm 0.010_{\text{syst}} \pm 0.032_{\text{ext}}) \times 10^{-5}$$

$$= (2.585 \pm 0.035) \times 10^{-5}$$

1.3% relative PDG 2012  $(2.2 \pm 0.4) \times 10^{-5}$  18% relative

$\text{BR}(\text{K}_{e4}) \times 10^5$



Error bars = sample dependent errors

# from BR to absolute form factor

Integrating 
$$d^3\Gamma = \frac{G_F^2 |V_{us}|^2}{2(4\pi)^6 m_K^5} \rho(S_\pi, S_e) J_3(S_\pi, S_e, \cos\theta_e) \times dS_\pi dS_e d\cos\theta_e.$$

One gets : 
$$\text{BR}(K_{e4}^{00}) = \tau_{K^\pm} \cdot |V_{us}|^2 \cdot f_s^2 \cdot \int d\Gamma_3 / (|V_{us}| \cdot f_s)^2$$

and using the kaon mean life time  $(1.2380 \pm 0.0021) \times 10^{-8} \text{ s}$

$$|V_{us}| \cdot f_s = 1.372 \pm 0.003_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.008_{\text{ext}}$$

corresponding to  $f_s = 6.092 \pm 0.012_{\text{stat}} \pm 0.017_{\text{syst}} \pm 0.045_{\text{ext}}$

for  $|V_{us}| = 0.2252 \pm 0.0009$

To be compared with  $f_s$  in the  $Ke4(+)$  mode :

$$f_s = 5.705 \pm 0.003_{\text{stat}} \pm 0.017_{\text{syst}} \pm 0.031_{\text{ext}}$$

*Phys.Lett. B715(2012) 105*

both values not really consistent within their errors: is there a missing ingredient ?

Preliminary!

# Summary

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Preliminary!

NA48/2 has collected a large sample of  $K_{e4}(00)$  events which leads to

- most precise BR value (1.3% relative precision) improving the world average value by more than one order of magnitude
- first form factor determination including significant dependence with  $q^2$  and  $S_e$  + evidence for rescattering effect in the final state
- absolute form factor value significantly away from the  $K_{e4}(+-)$  corresponding form factor value
- results are PRELIMINARY but should be finalized by Summer 2013 and go for publication
- discussion with theory groups is most important for a correct and precise formulation of the processes under study.
- Prospects : both  $K_{\mu 4}$  modes should be also accessible with  $O(10^3)$  events,  $K_{\mu 4}(+-)$  known so far from 9 events,  $K_{\mu 4}(00)$  never observed ...



# Spares

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# $f_s(0,0) = f_s$ measurement systematic uncertainties

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$$f_s = 6.092 \pm 0.012_{\text{stat}} \pm 0.017_{\text{syst}} \pm 0.045_{\text{ext}}$$

Preliminary!

Source	relative contribution (%)
BR( $K_{e4}$ ) statistical error	0.19
BR( $K_{e4}$ ) systematic error	0.19
Form factor energy dependence (systematic error)	0.10
Form factor cusp effect (systematic error)	0.13
Integration method (systematic error)	0.02
Radiative effects in integration (systematic error)	0.03
Total experimental error	0.32
BR( $K_{e4}$ ) external error	0.62
Kaon lifetime (external error)	0.08
$ V_{us} $ (external error)	0.40
Total error (including external errors)	0.81