Theory of rare and radiative kaon decays

Giancarlo D'Ambrosio
INFN-Sezione di Napoli

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- $K_S(K_L) \to \mu\mu$
- $K_L \to \pi^0 e^+ e^-$, the related channels $K \to \pi\gamma\gamma$ and $K_S \to \pi^0 e^+ e^-$ Buchalla, *et al.*
- CP violation in $K \to \pi\pi\gamma$, $K \to \pi\pi ee$ Cappiello, Cata,G.D.,Gao
- Conclusions

$K_S(K_L) \to \mu\overline{\mu}$

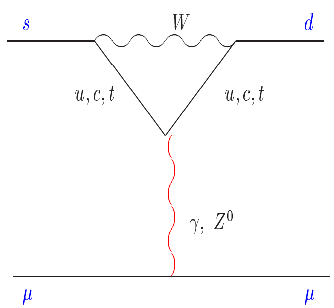
- $A(K^0 \to l^+ l^-) = \bar{u}_l(iB \ + \ A\gamma_5)v_l$

- if No polarization is measured

$$\Gamma(K_{L,S} \to \mu^+\mu^-) = \frac{m_K \ \beta_l}{8\pi} \left(|A|^2 \ + \ |B|^2\right) \qquad \beta = \sqrt{1 - \frac{4m_\mu^2}{m_K^2}}$$

- For Polarization studies see Peter Heczeg Phys.Rev. D27 (1983) 1512 LA-UR-82-3262

$K_L \to \mu\overline{\mu}$



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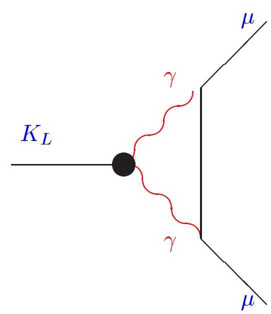


$$\frac{\Gamma(K_L \to \mu\overline{\mu})}{\Gamma(K_L \to \gamma\gamma)} = |ReA|^2 + |ImA|^2 = (1.238 \pm 0.024) \cdot 10^{-5}$$

$$|ImA|^2 = 27.14\ r \Longrightarrow \quad |Re(A_{\rm SD} + A_{\rm LD})|^2 = (0.98 \pm 0.55)\ r$$

r kin. factor

Good determination of $A_{\rm LD} \Rightarrow A_{\rm SD} \Rightarrow$ SM test

Low energy form factor + matching with VMD to QCD

We use a QCD motivated form factor G.D. Isidori, Portoles;Isidori, Unterdorfer

$$A(K_L \to \gamma^*(q_1)\gamma^*(q_2)) = A_{\gamma\gamma}^{\exp}\left[1 + \alpha\left(\frac{q_1^2}{q_1^2 - M_V^2} + \frac{q_2^2}{q_2^2 - M_V^2}\right) + \beta\frac{q_1^2 q_2^2}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)}\right]$$

α fixed from $K_L \to \gamma\gamma^*$ β from $K_L \to e^+e^-\mu\bar{\mu}$ (not yet)

Matching with QCD $q_1^2, q_2^2 \to \infty$ imposes $|1 + 2\alpha + \beta| \simeq 0.3$

Isidori, Unterdorfer

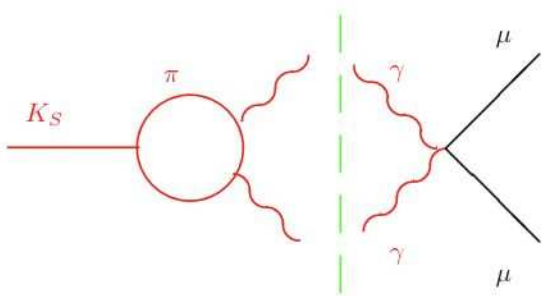
- $|ImA|^2 = 27.14\ r \Longrightarrow \quad |Re(A_{\rm SD} + A_{\rm LD})|^2 = (0.98 \pm 0.55)\ r$

  r kin. factor

- By very accurate study they obtain $-3.1 < A_{\rm SD} < 1.7 \Longrightarrow B(K_L \to \mu\mu)_{SD} < 2.5 \times 10^{-9}$

$K_S \to \mu\overline{\mu}$ Ecker,Pich; Isidori, Unterdorfer

- $A(K^0 \to l^+ l^-) = \bar{u}_l (iB \ + \ A\gamma_5) v_l$
- $\Gamma(K_{L,S} \to \mu^+\mu^-) = \frac{m_K \ \beta_l}{8\pi} \left(|A|^2 \ + \ |B|^2 \right)$ $\qquad \beta = \sqrt{1 - \frac{4m_\mu^2}{m_K^2}}$
- Short distance: **ONLY CP Violating from A**
- SM $B(K_S \to \mu\overline{\mu})_{SD} = 1 \times 10^{-5} |\Im(V_{ts}^* V_{td})|^2 \sim 10^{-13}$; NP $\sim 10^{-11}$ allowed;
- LD VERY ACCURATE 5×10^{-12}, error from expt $B(K_S \to \gamma\gamma)$
- LHCB $B(K_S \to \mu\overline{\mu}) < 11 \times 10^{-9}$ at 95% CL after 40 years


K_S
π
γ
γ
μ
μ

$$K_L(p) \to \pi^0(p_3)\gamma(q_1)\gamma(q_2)$$

$$\text{Lorentz + gauge invariance} \quad \Rightarrow \quad M \sim \quad A(y,z) \qquad B(y,z)$$

		$A(y,z)$	$B(y,z)$
		$\gamma\gamma$	$\gamma\gamma$
$y = p\cdot(q_1-q_2)/m_K^2, \quad z=(q_1+q_2)^2/m_K^2$		$J=0$	D − wave too
$r_\pi = m_\pi/m_K$		$F^{\mu\nu}F_{\mu\nu}$	$F^{\mu\nu}F_{\mu\lambda}\partial_\nu K_L \partial^\lambda \pi^0$

- $\frac{d^2\Gamma}{dydz} \sim z^2|A+B|^2 + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2$ $\qquad S, B$
- Different gauge structure ⇒ B≠0 at $z \to 0$ (collinear photons).

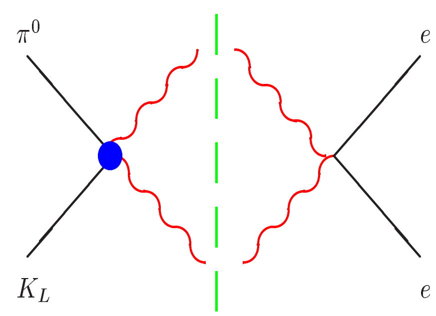
Crucial role in $K_L \to \pi^0 e^+ e^-$

A suppressed by m_e/m_K

B is not

Morozumi et al, Flynn Randall

Sehgal Heiliger, Ecker et al., Donoghue et al.

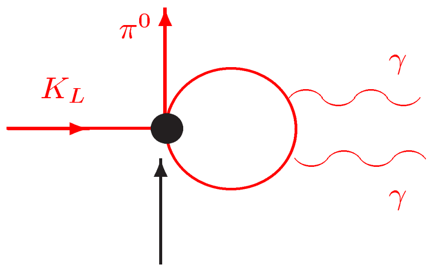


$K_L \to \pi^0 \gamma\gamma$

- $O(p^4)$ Ecker, Pich, de Rafael; Cappiello,G.D

CT Loop

0



only A **But** $\frac{\Gamma(K_L \to \pi^0 \gamma\gamma)_{p_4}}{\Gamma(K_L \to \pi^0 \gamma\gamma)_{\text{exp}}} \sim \frac{1}{2.5}$

G_8 from $K_S \to 2\pi$

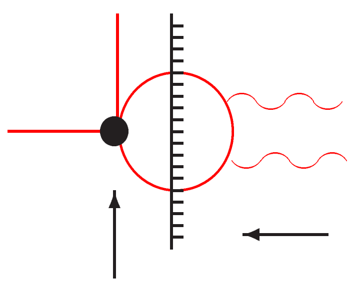
- $O(p^6)$ A, B from:

3 CT′s

$F_{\mu\nu} F^{\mu\alpha} \partial_\alpha K_L \partial^\nu \pi^0$

$F^2 \ \partial K_L \partial \pi^0$

$F^2 \ m_K^2 K_L \pi^0$



Cappiello, G.D., Miragliulo
Cohen, Ecker, Pich

← Full description of unitarity cut

$$A(K \to 3\pi) = a + b\ Y + c\ Y^2 + d\ X^2$$

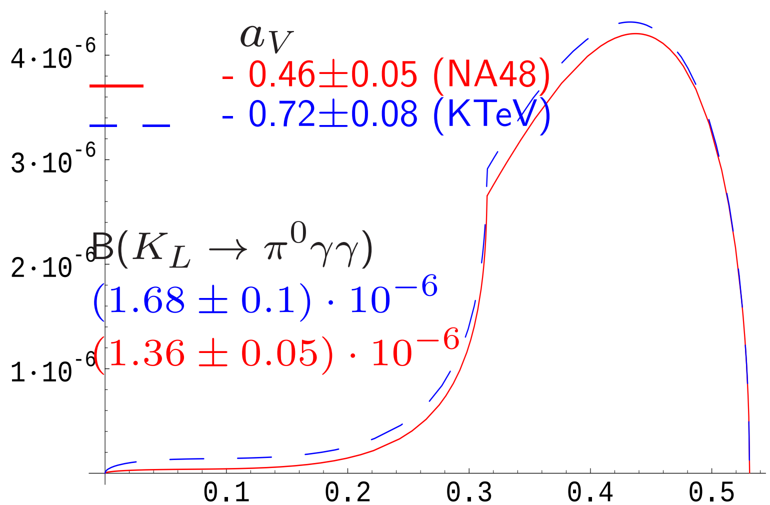
$$A_{\rm CT} = \alpha_1(z - r_\pi^2) + \alpha_2$$
$$B_{\rm CT} = \beta$$

VMD $\Rightarrow$ 1 coupling a_V $(\sim -0.6$ G.D., Portoles$)$

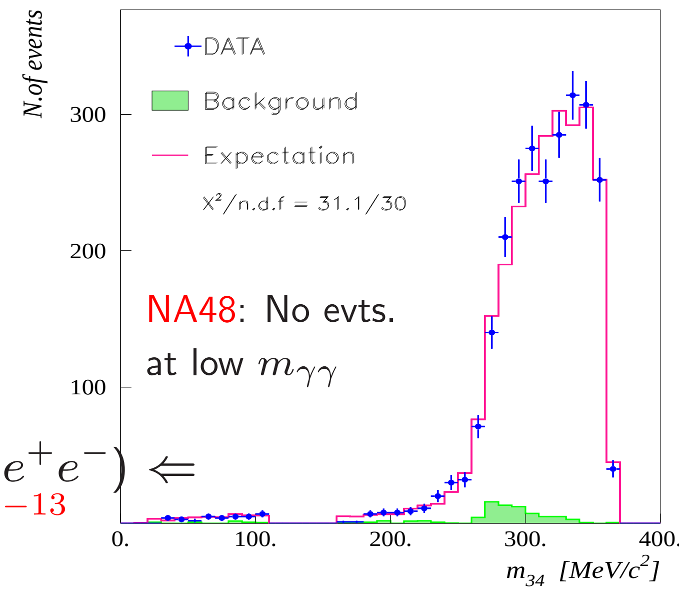
(Ecker, Pich, de Rafael;Sehgal et al.)

$$\alpha_1 = \frac{\beta}{2} = -\frac{\alpha_2}{3} = -4a_V \sim 2 \qquad \text{n.d.a.}\sim 0.2$$

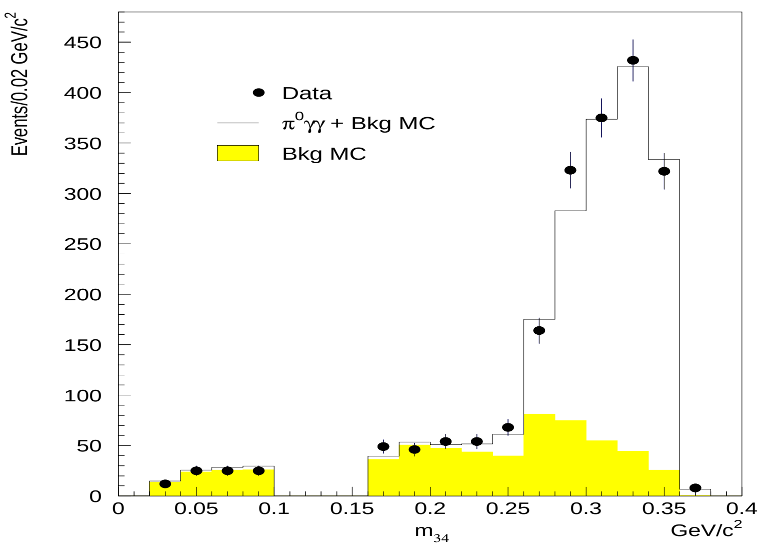
- KTeV and NA48: 1 parameter fit (a_V) with all the unitarity corrections



$B(K_L \to \pi^0 e^+e^-) \Leftarrow$
$< 5\cdot 10^{-13}$



$K_L \to \pi^0\gamma\gamma$: KTeV recent data



$$B = (1.29 \pm 0.03 \pm 0.05) \times 10^{-6}$$

$$a_V = -0.31 \pm 0.05 \pm 0.07$$

$K^+ \to \pi^+\gamma\gamma$

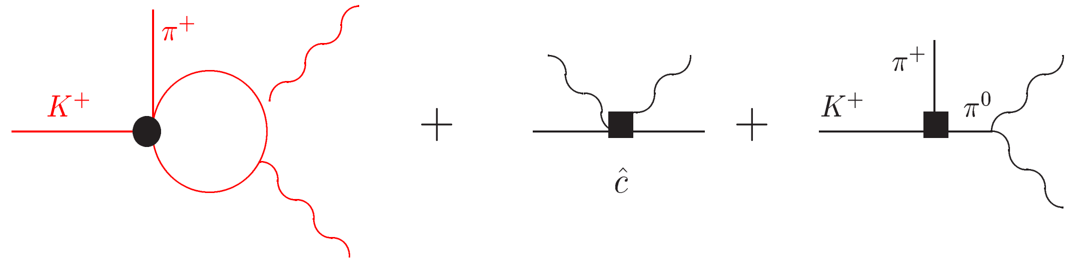
$$\gamma\gamma \quad \text{in} \quad \overbrace{\begin{matrix} F_{\mu\nu}F^{\mu\nu} & F\tilde{F} \\ P=+1 & P=-1 \\ A & C \end{matrix}}^{J=0} \qquad \begin{matrix} J=2 \\ \\ \\ B \end{matrix} \quad + \ \ldots$$

Lorentz + gauge invariance

$$\frac{d^2\Gamma}{dydz} \sim \left[z^2(|A+B|^2+|C|^2) + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2\right]$$

$K^+ \to \pi^+\gamma\gamma$

- $O(p^4)$



Ecker, Pich, de Rafael

In factorization $\hat{c} = \frac{128\pi^2}{3}[3(L_9 + L_{10}) + N_{14} - N_{15} - 2N_{18})] = 2.3(1-2k_f)$

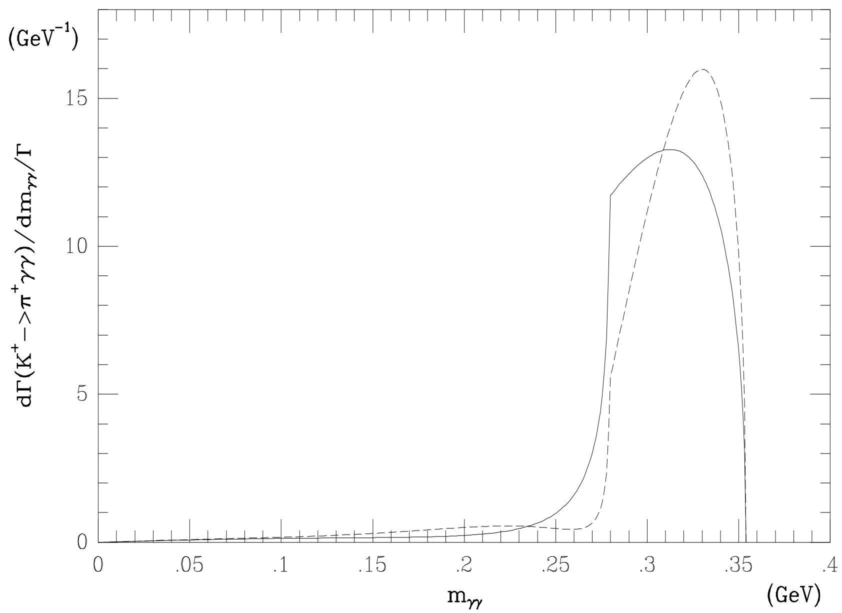
spin-1 contributions (axials) to $\hat{c}$

- $O(p^6)$ G.D., Portoles 96

Unitarity corrections: 30%-40%

a_{V+} negligible

$K^+ \to \pi^+\gamma\gamma$



	$\hat{c}$
———	0
— — —	-2.3

BNL 787 (96) got 31 events

i) confirm $O(p^6)$

ii) Br$\sim$ $(6 \pm 1.6) \cdot 10^{-7}$

iii) $\hat{c} = 1.8 \pm 0.6$

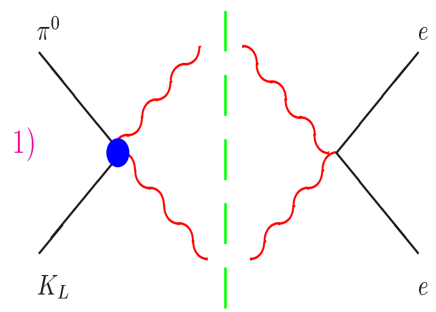
E949 no events at low $m_{\gamma\gamma}$:

NA48 published $K^+ \to \pi^+ e^+ e^- \gamma$ $\hat{c} = 0.90 \pm 0.45$

NA48 preliminary $B = (1.07 \pm 0.04 \pm 0.08) \cdot 10^{-6}$, $\hat{c} = 2 \pm 0.25$

$K_L \to \pi^0 e^+ e^-$: summary

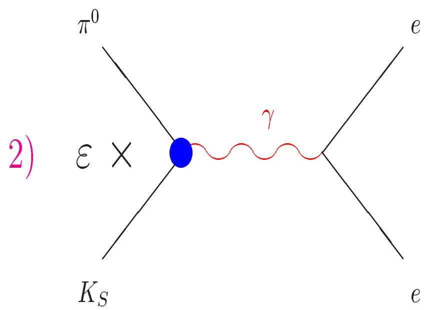
$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \leq 2.8 \cdot 10^{-10}$ at 90% CL KTeV

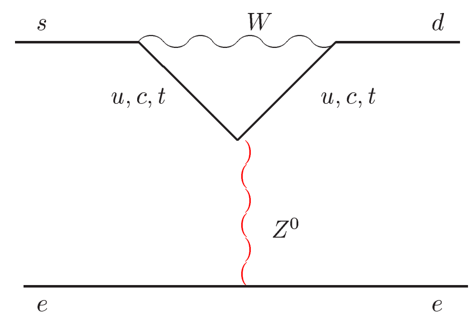


CP conserving NA48

Br$(K_L \to \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$

V-A$\otimes$ V-A $\Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle$ violates CP





↑ $\mathrm{B}(K_s \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3\, a_S^2 - 6.8 \frac{Im\lambda_t}{10^{-4}}\, a_S + 2.8 \left(\frac{Im\lambda_t}{10^{-4}}\right)^2\right] \cdot 10^{-12}$$

$$[17.7 \pm \quad 9.5 + \quad 4.7] \quad \cdot 10^{-12}$$

$$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance $\Rightarrow$ Electric (E) and Magnetic(M) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarized photons

$$\frac{d^2\Gamma}{dz_1dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2Re(E^*_{IB}E_D) + |E_D|^2$$

$$\downarrow$$

Low Theorem $\Rightarrow E_{IB} \sim \frac{1}{E^*_\gamma} +$ c $\qquad E_D$, M chiral tests

We need FIGHT DE/IB$\sim 10^{-3}$

	IB	DE_{exp}	
$K_S \to \pi^+\pi^-\gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	10^{-4} $(\Delta I = \frac{3}{2})$	$(0.6 \pm 0.04)10^{-5}$ PDG	$M1, E1$
$K_L \to \pi^+\pi^-\gamma$	10^{-5} (CPV)	$(2.84 \pm 0.11)10^{-5}$ KTeV	$M1$, VMD

CPV is only from IB K_L (also measured in $K_L \to \pi^+\pi^-e^+e^-$)

BUT IB suppressed in K^+ and K_L.

$K^+ \to \pi^+\pi^0\gamma$

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

$E1$ and $M1$ are measured with Dalitz plot

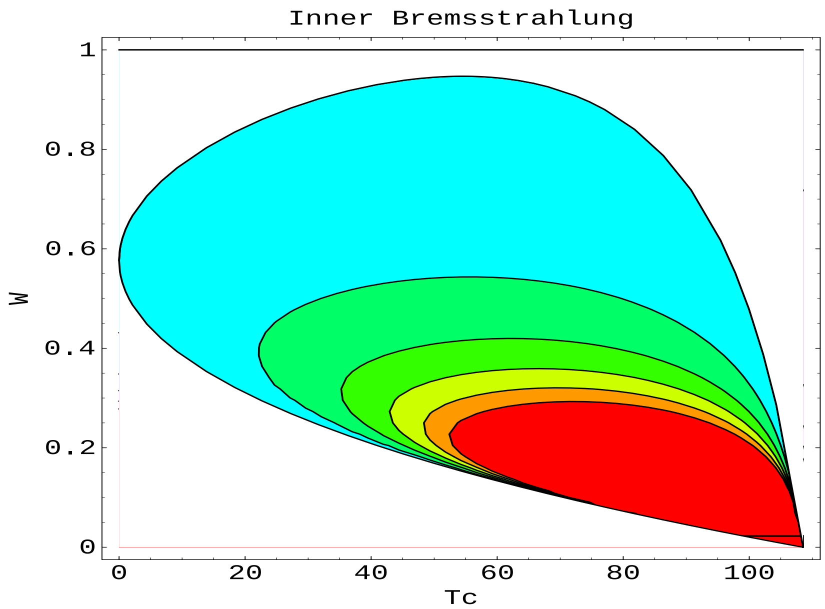
$$\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^*\partial W^2}\left[1 + \frac{m_{\pi^+}^2}{m_K}2Re\left(\frac{E1}{eA}\right)W^2 + \frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2\right)W^4\right]$$

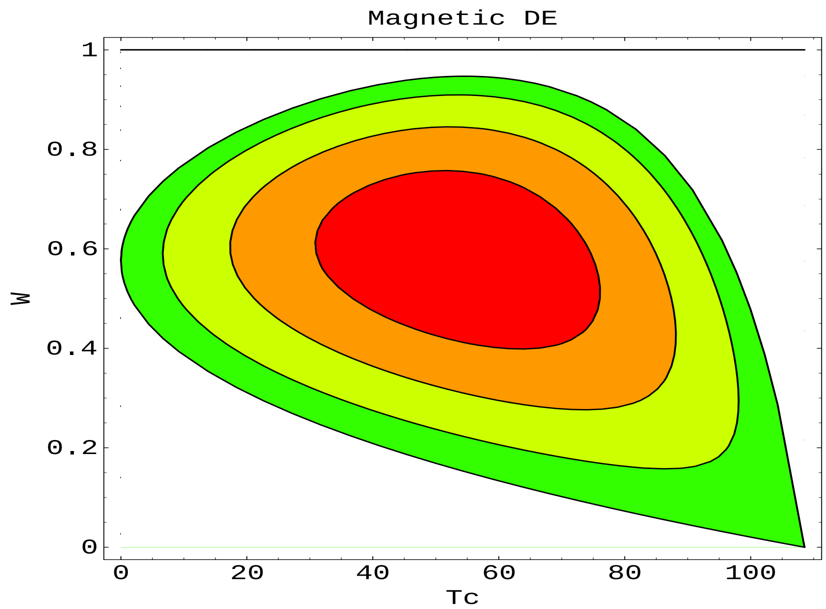
$$W^2 = (q\cdot p_K)(q\cdot p_+)/(m_\pi^2 m_K^2)$$

$$A = A(K^+ \to \pi^+\pi^0)$$

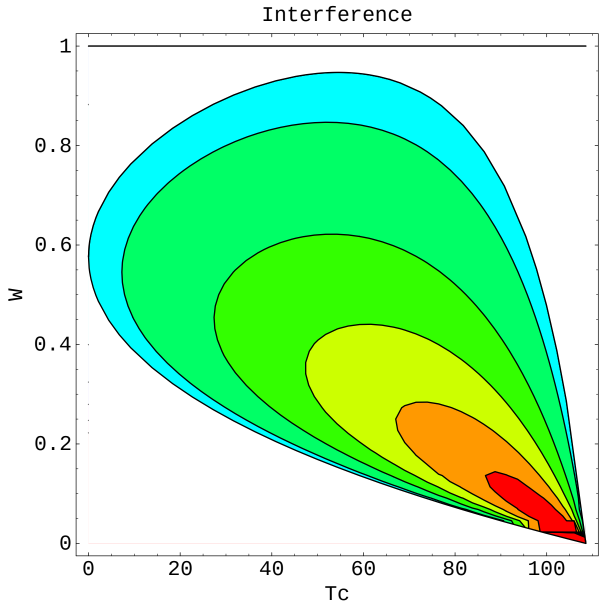
$K^+ \to \pi^+\pi^0\gamma$ $W - T_c$ **Dalitz plot**

Integrating over T_c deviations from IB measured





CP asymmetry $K^+ \to \pi^+\pi^0\gamma$



Dalitz plot analysis crucial

SM $\leq \mathcal{O}(10^{-5})$ Paver et al.

NP $\leq \mathcal{O}(10^{-4})$ Colangelo et al.

NA48/2 $< 1.5 \cdot 10^{-3}$ at 90% CL

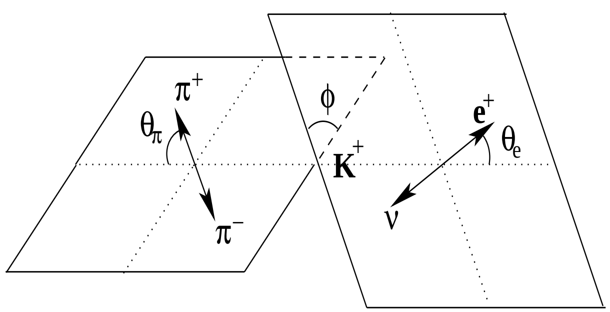
BUT NOT in the interesting interf. kin. region (statistics)

K_{l4} and $\pi\pi$ strong phases $\delta^l_I(s)$

Cabibbo Maksymowicz

$$\frac{G_F}{\sqrt{2}} V_{us}\ \bar{e}\gamma^\mu(1-\gamma^5)\nu\ H_\mu(p_1,p_2,q)$$

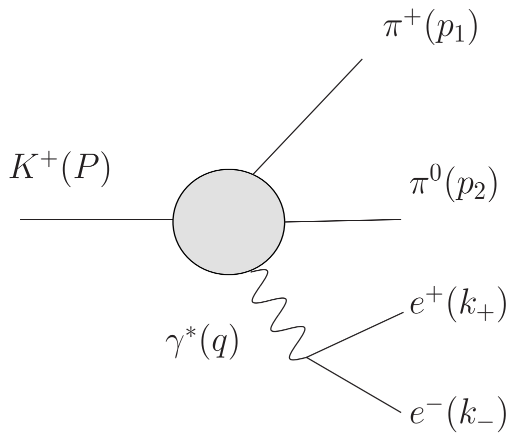
$$H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\ \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta . \qquad F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$



- crucial to measure $\sin\delta \Longrightarrow$ interf F_3
- Look angular plane asymmetry

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$

Sehgal et al; Savage,Wise et al

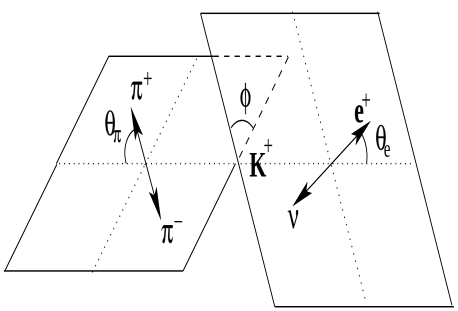


- $\mathcal{M}_{LD} = \frac{e}{q^2}\, \bar{e}\gamma^\mu(1-\gamma^5)\; e\; H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\, \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

- Interference $E \quad M$ novel compared to $K_L \to \pi^+\pi^-\gamma$
- $E \quad M$ known from $K_L \to \pi^+\pi^-\gamma$ (IB and DE)

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$ θ_ℓ, ϕ **vs** θ_π

$$\frac{d^5\Gamma}{dE^*_\gamma dT^*_c dq^2 d\cos\theta_\ell d\phi} = \mathcal{A}_1 + \mathcal{A}_2 \sin^2\theta_\ell + \mathcal{A}_3 \sin^2\theta_\ell \cos^2\phi$$
$$+\mathcal{A}_4 \sin 2\theta_\ell \cos\phi + \mathcal{A}_5 \sin\theta_\ell \cos\phi + \mathcal{A}_6 \cos\theta_\ell$$
$$+\mathcal{A}_7 \sin\theta_\ell \sin\phi + \mathcal{A}_8 \sin 2\theta_\ell \sin\phi + \mathcal{A}_9 \sin^2\theta_\ell \sin 2\phi$$



- $\mathcal{A}_{1,..,4}$ IB, $\mathcal{A}_{8,9}$, CPV B-M interf.
- $\dfrac{\Re(E_B\ M^*)}{|E_B|^2 + |M|^2}$ is maximal, ~ 0.1
- $\mathcal{A}_{5,6,7}$ interf. axial leptonic current , $\mathcal{A}_P$ SD

$K^+ \to \pi^+\pi^0\gamma^* \to \pi^+\pi^0 e^+ e^-$

Cappiello, Cata,G.D. and Gao,

- the asymm. , $\frac{\Re(E_B\ M^*)}{|E_B|^2+|M|^2}$, not as lucky $E_B >> M$:

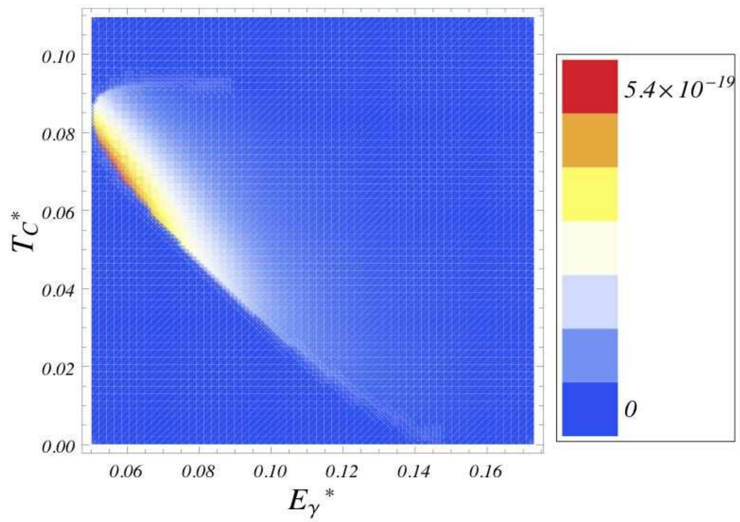
- $B(K^+)_{IB} \sim 3.3\times 10^{-6} \ \sim \ 50\ B(K^+)_M$

- Short distance info without having simultaneously K^+ and K^-, asymm. in phase space, (P-violation) interesting! No ϵ-contamination

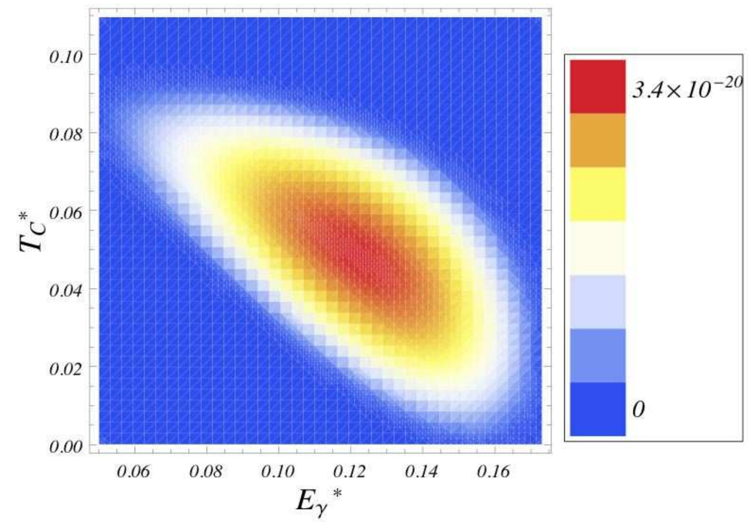
- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B

- at $q^2 = 50\mathrm{MeV}$ IB only 10 times larger than DE

q_c (MeV)	B $[10^{-8}]$	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44



IB



DE

How to extract SD from $K^+ \to \pi^+\pi^0 e^+ e^-$

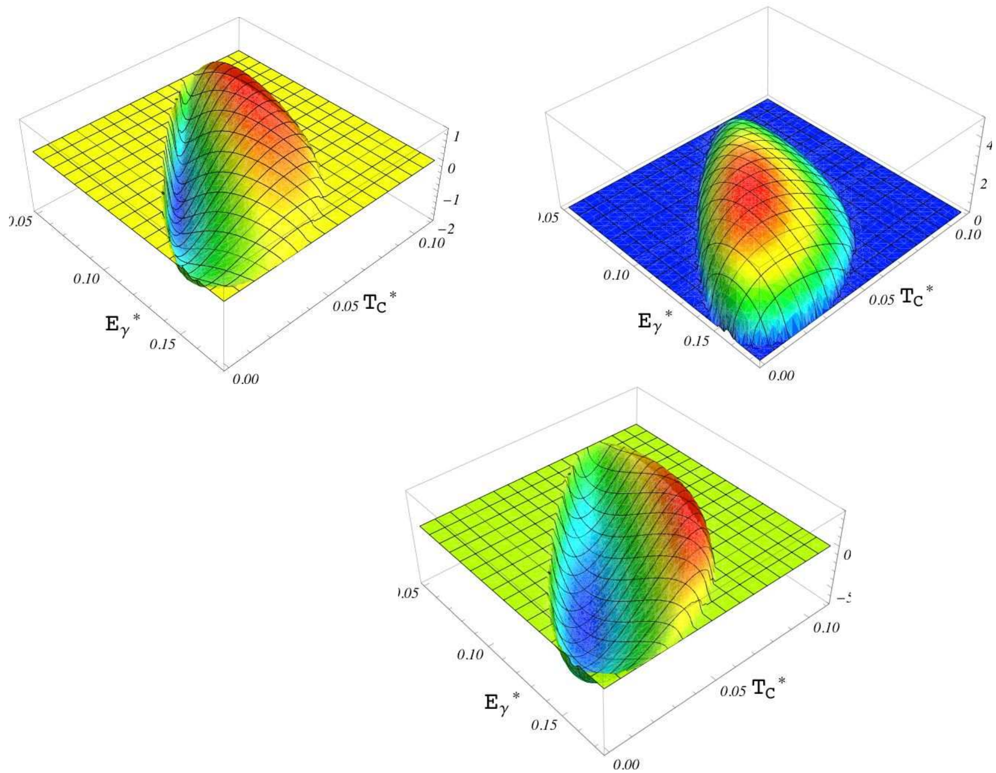
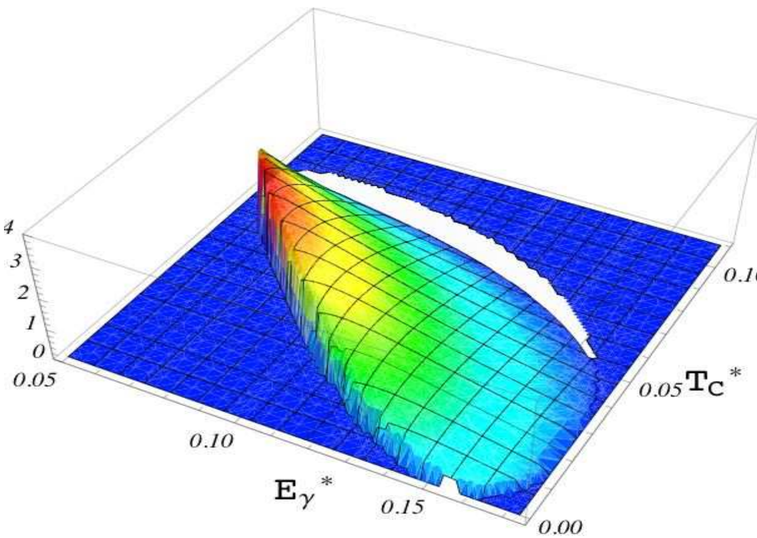
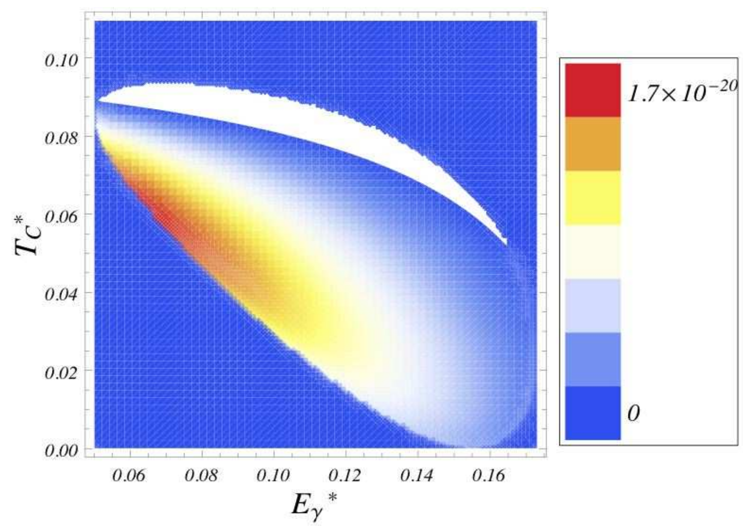


Figure 1: Dalitz plot for the contributions to $A_P^{(S)}$ (in arbitrary units) at $q^2 = (50 \text{ MeV})^2$.

Novel CP violation contributions (compared to $A_{CP}(K^+ \to \pi^+\pi^0\gamma)$)

$$A_{CP} = \frac{\Gamma(K^+ \to \pi^+\pi^0 e^+e^-) - \Gamma(K^- \to \pi^-\pi^0 e^+e^-)}{\Gamma(K^+ \to \pi^+\pi^0 e^+e^-) + \Gamma(K^- \to \pi^-\pi^0 e^+e^-)}$$





Conclusions

- CP conserving contribution in $K_L \to \pi^0 ee$ is small: confirmed also from $K^+ \to \pi^+\gamma\gamma$ by NA48/2 - NA62

- $K^+ \to \pi^+\pi^0 ee$ interesting new channel , see also $K^+ \to \pi^+\pi^0\mu\mu$

- LHCB entering in the scene, 40 years step, see also other K_S channels